

CEGE 4501 Hydrologic Design

Chapter 4: Evapotranspiration



UNIVERSITY OF MINNESOTA
Driven to DiscoverSM

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Introduction I

Evapotranspiration (ET) is the processes by which liquid water at the Earth's surface is transformed into water vapor. Direct transformation from liquid to vapor over a water surface or bare soil is called **evaporation**, while vaporization of water due to a plant's metabolism and growth is called **transpiration**.

ET, though a relatively slow flux, is one of the most important elements of the hydrologic water cycle. ET controls water mass and energy transfer within **land-vegetation-atmosphere** continuum. Additionally, ET contributes significantly to **freshwater losses** in water resources and agricultural systems. Therefore, effective management of ET can significantly improve water and food security.

ET is primarily a water vapor **mass flux** and thus its measurement is not trivial, especially at the large scale. ET requires the following ingredients to occur:

- An energy source (e.g., Sun's radiative energy, wind kinetic energy)
- Liquid/solid water (e.g., lakes, soil moisture, snow-covered surfaces, etc.)
- A transport mechanism (e.g., molecular diffusion, convection, conduction)



Figure 1: Schematic of ET (left), shade balls dumped into a reservoir to mitigate evaporation (middle) and a dried agricultural field due to excessive evaporation (right).

Heat Transfer in Hydrology I

Sensible Heat

It is now time to expand on the surface energy balance equation, we covered at the end of Chapter 3. As you recall, the **net radiation (R_n)** at the surface can be partitioned as:

$$R_n = LE + H + G$$

where $LE(\lambda E)$ is the **latent heat flux**, H is the **sensible heat flux**, and G is the **ground heat flux**.

The **sensible heat** is the part of a substance's (water) internal energy that is **proportional to its temperature**, hence we can sense it. Sensible heat exchange between a substance (system) and its surroundings (environment) **changes its temperature**. In terms of the land surface energy budget, sensible heat is the energy transfer largely via convection.

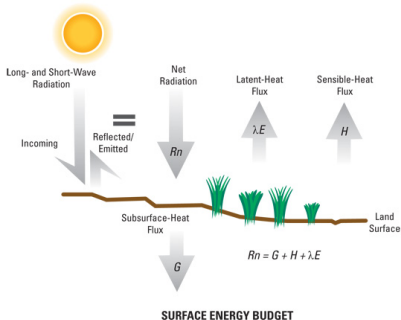


Figure 2: Schematic of the surface energy budget (SEB) (Credit: USGS).

Heat Transfer in Hydrology II

The total heat exchange for a system due to a temperature change is defined as:

$$dQ = c_p m dT$$

m : mass [kg] dT : Temp change [K] c_p : specific heat [$\text{J kg}^{-1} \text{K}^{-1}$]

Notice that the specific heat is the constant of proportionality as we discussed. In practice, for hydrologic applications, we use the **specific heat at constant pressure** since the atmospheric pressure changes little over small distances.

Latent Heat

The energy needed to change the phase of a substance (solid, liquid, vapor) without any change in its temperature is called the **latent heat** of phase change. Here, are typical values we need for hydrologic applications:

Latent Heat of Fusion: Melting from solid to liquid $\mathcal{L}_{s\ell} = 3.34 \times 10^5 \text{ [J/kg]}$

Latent Heat of Vaporization: Change from liquid to water vapor $\mathcal{L}_{\ell v} = 2.50 \times 10^6 \text{ [J/kg]}$

Latent Heat of Sublimation: Change from ice to vapor $\mathcal{L}_{iv} = 2.85 \times 10^6 \text{ [J/kg]}$

Primarily, as hydrologists we are interested in $\mathcal{L}_{\ell v}$ for ET, which is dependent on temperature as described below:

$$\mathcal{L}_{\ell v} = 2.5 \times 10^6 - 2370T \text{ [J/kg]}$$

where T [$^{\circ}\text{C}$] is temperature in Celsius. The **sensible and latent heat transfer** mechanisms are the two *most important* components of heat energy exchange in hydrologic systems and applications. It is important to note that it requires 4200 [Joule] of heat energy to increase the temperature of 1 [kg] of water by one degree Celsius. However, we need 2.5×10^6 [Joules] of heat energy to evaporate 1 [kg] of water.

Transport Mechanisms I

Diffusion or Conduction: Diffusion of water mass or conduction of heat or momentum are due to **random molecular motion**. The transfer occurs from areas of higher concentration of the variable of interest to areas of lower concentration. Heat, momentum and mass molecular transport mechanisms are typically explained via **conduction, viscosity, and diffusion coefficients**, respectively.

Advection: Transfer of heat, momentum or mass due to the **bulk motion of fluid parcels**.

Turbulent Diffusion: Turbulence is irregular swirls of fluid motion. In atmosphere, it is mainly generated by **mechanical shear due to wind velocity or thermally driven buoyant forces**, leading to heat, mass, and momentum transfer. Turbulent diffusion is similar to molecular diffusion; however, in turbulent diffusion we are dealing with random motion of the fluid parcels beyond the effects of molecular diffusion.

Convection: Combination of advection and turbulent diffusion in transfer of heat, mass and momentum.

Figure 3: Schematics of shear turbulence (left) and buoyant turbulence (right) driving heat, moisture and momentum transfer at the Earth's surface (Credit: The COMET Program).

Transport Mechanisms II

The amount of heat, moisture, and momentum transport in the atmosphere are measured by their fluxes. **A flux is the transfer of a quantity per unit area per unit time.**

Advective Fluxes: As mentioned, advection is the transport of heat, mass, and momentum by bulk fluid motion due to its velocity.

Mass Flux: ρu	$[\text{kg m}^{-2} \text{ s}^{-1}]$
Momentum Flux: ρu^2	$[\text{N m}^{-2}]$
Heat Flux: $\rho_a c_p T u$	$[\text{W m}^{-2}]$
Moisture Flux: $\rho_a q_v u$	$[\text{kg m}^{-2} \text{ s}^{-1}]$
Moisture-Energy Flux: $\mathcal{L}_{\ell v} \rho_a q_v u$	$[\text{W m}^{-2}]$

where ρ_a [kg m^{-3}] is air density, u [m s^{-1}] is the average streamwise velocity, T [K] is the temperature, q_v [kg water/kg air] is the specific humidity and $\mathcal{L}_{\ell v}$ [J kg^{-1}] is the latent heat of vaporization.

Diffusive and Turbulent Fluxes: For ET, we are primarily interested in convective transport of heat, mass and momentum through the **turbulent diffusion**. **First order** approximation of the turbulent fluxes can be expressed similar to the way we quantify molecular diffusion in the sense that the fluxes are proportional to the **gradient of the quantity** of interest. Below are the diffusive and turbulent fluxes for momentum, heat and mass.

Transport Mechanisms III

Momentum Flux

Newton's Law of Viscosity: $\tau = -\mu \frac{du}{dz} = -\rho_a \nu \frac{du}{dz}$ [N m⁻²]

τ : momentum flux (shear stress) [N m⁻²]

μ : dynamic viscosity [N s m⁻²]

u : velocity [m s⁻¹]

ρ_a : air density [kg m⁻³]

ν : kinematic viscosity [m² s⁻¹]

z : distance [m]

$\frac{du}{dz}$: vertical velocity gradient [s⁻¹]

Turbulent Momentum Flux or Reynold's Stress: $\tau = -\rho_a K_M \frac{du}{dz}$ [N m⁻²]

K_M : Momentum eddy diffusivity [m² s⁻¹]

Note: $K_M \gg \nu$

Transport Mechanisms IV

Heat Flux

Fourier's Law of Conduction: $H = -D_H \frac{dT}{dz} = -\rho_a c_p \alpha \frac{dT}{dz}$ [W m⁻²]

D_H : thermal conductivity [W m⁻¹K⁻¹]

T : temperature [K]

z : distance [m]

c_p : specific heat [J kg⁻¹ K⁻¹]

α : thermal diffusivity [m²s⁻¹]

ρ_a : air density ≈ 1.3 [kg m⁻³]

Turbulent Sensible Heat Flux: $H = -\rho_a c_p K_H \frac{dT}{dz}$ [W m⁻²]

K_H : thermal eddy diffusivity [m² s⁻¹]

Note: $K_H \gg \alpha$

Transport Mechanisms V

Moisture Flux

Fick's Law of Molecular Diffusion: $E = -\rho_a D_E \frac{dq_v}{dz}$ [kg m⁻² s⁻¹]

ρ_a : air density [kg m⁻³]

D_E : molecular diffusivity of water vapor in air [m² s⁻¹]

q_v : specific humidity [kg water/kg air]

z : distance [m]

Turbulent Evaporation or Moisture Flux: $E = -\rho_a K_E \frac{dq_v}{dz}$ [kg m⁻² s⁻¹]

K_E : moisture eddy diffusivity [m² s⁻¹]

Note: $K_E \gg D_M$

As we have discussed the **turbulent diffusivity coefficients are much larger than the various molecular diffusion coefficients**. This is true, especially in the atmospheric boundary layer, and thus we typically neglect the molecular diffusion portion of transport.

Some experimental evidence suggests that in a neutral atmosphere (no density gradient) $K_H = K_E = 1.35K_M$, which will be defined in the next slide. However, often in practice these three diffusivity values are assumed to be equal.

ABL Stability I

Hydrologic evapotranspiration fluxes are largely due to **turbulent heat and moisture fluxes** in the **atmospheric boundary layer (ABL)**.

ABL: The lowest part of the atmosphere where the air flow properties are highly affected by the earth's surface. The depth of the ABL is typically around 1-2 km.

Neutral ABL: The air density is uniform throughout the ABL depth, meaning if we displace a parcel of air, it will stay in its new position.

Stable ABL: Denser air is below less-dense air through the entire ABL. If we displace an air parcel, it tends to return to its original position.

Unstable ABL: Less-dense air is below denser air. If we displace a parcel of air, it may not return to its original position.

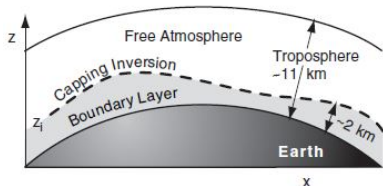


Figure 4: A schematic of ABL (Credit: Stull, 2015).

ABL Stability II

Why is stability of ABL important?

When the atmosphere is **stable**, the turbulent moisture and heat fluxes are **suppressed**, whereas in **unstable** atmospheres, they are **enhanced**. Therefore, **ET is a function of atmospheric stability condition**.

There are two main turbulence mechanisms that drive ABL instability:

Static Instability: Buoyancy driven turbulence (buoyancy \gg wind shear kinetic energy)

Dynamic Instability: Wind shear driven turbulence (wind shear \gg buoyancy kinetic energy)

How can we determine ABL stability?

There are several dimensionless parameters that are commonly used to define atmospheric stability from data. The **Gradient Richardson Number** is a dimensionless parameter consisting of the ratio of the buoyant turbulent and mechanical shear turbulence productions:

$$R_i = \frac{\frac{g}{T_v} \cdot \frac{\partial T_v}{\partial z}}{\left[\left(\frac{\partial \bar{u}}{\partial z} \right)^2 \right]} = \frac{\text{Buoyant turbulence production}}{\text{Shear turbulence production}}$$

where z denotes vertical direction, T_v is the virtual temperature, g is gravitational acceleration, and \bar{u} is the average wind velocity in the wind direction. The values of R_i are interpreted as

Statically Unstable: $R_i < 0$ **Neutral Condition:** $R_i = 0$ **Statically Stable:** $R_i > 0$

ABL Stability III

Another important stability parameter in the ABL is the so-called **Obukhov Length L** :

$$L = \frac{-\overline{T}_v u_*^3}{\kappa g \left(\frac{H}{c_p \rho_a} \right)} \quad [\text{m}]$$

where \overline{T}_v is the mean virtual temperature, $u_* = \sqrt{\frac{\tau}{\rho_a}}$ is the shear velocity, where τ is the shear stress [N m^{-2}] in an arbitrary layer of the air and ρ_a denotes the air density, $\kappa \simeq 0.41$ is the Von Karman constant, g is the earth gravitational acceleration and H is the sensible heat flux at the surface.

The Obukhov length (L) is an important scaling variable used to account for the effects of atmospheric stability condition on momentum, heat and mass fluxes. Typically, we divide the height above the surface by L to define a dimensionless representation of the Obukhov length:

$$\zeta = \frac{z}{L},$$

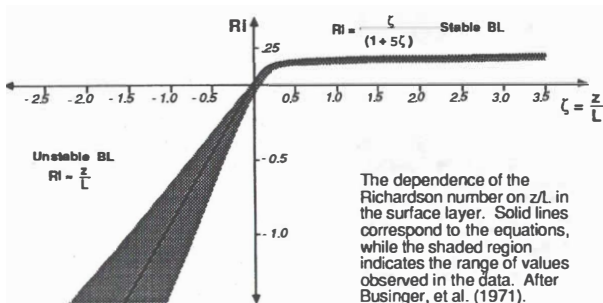
where ζ is often called the **surface layer height scaling variable**.

When the Obukhov length is negative $L < 0$ the heat flux is upward and thus the atmosphere is unstable and when it is positive $L > 0$, the heat flux is downward and thus the atmosphere is stable.

The gradient Richardson number R_i and ζ are related to each other through the following semi-empirical relationship:

ABL Stability IV

$$\zeta = \frac{R_i}{1 - 5R_i} \quad \text{stable } (\zeta > 0; 0 \leq R_i < 0.2)$$
$$\zeta = R_i \quad \text{unstable } (\zeta < 0; R_i < 0)$$



Before we move forward, it is important to clarify that the discussion is confined to the [Atmospheric Surface Layer \(ASL\)](#). This layer is typically taken as the [bottom 10% of the ABL](#) where the vertical velocity, temperature, and moisture vary rapidly, while vertical fluxes of heat, moisture and momentum are approximately constant.

ASL Wind Profile I

Log Law in Neutral ABL

In order to understand ET fluxes from the Earth's surface to the ASL, we must have an understanding of the **mean wind velocity profile** as it is the main driver of the turbulent diffusive fluxes. It has been (theoretically) shown that the average velocity profile in a **neutral** ASL follows a logarithmic shape as follows:

$$\bar{u} = \frac{u_*}{\kappa} \ln \left(\frac{z}{z_0} \right).$$

where z_0 [m] is the **momentum roughness length**, u_* [m s^{-1}] is the **friction velocity**, and $\kappa \simeq 0.41$ is the **Von Karman constant**. Additionally, over a canopy, where the roughness elements are densely packed, the wind velocity becomes theoretically zero at height $d + z_0$, where d is called the **zero-plane displacement height** (Figure 5). As a result, we have

$$\bar{u} = \frac{u_*}{\kappa} \ln \left(\frac{z - d}{z_0} \right).$$

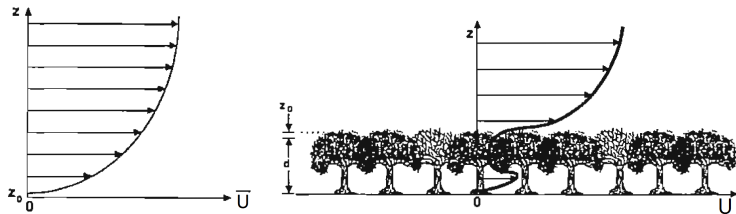


Figure 5: Typical logarithmic variation of a vertical wind profile in neutral ASL over an almost flat (left) and densely packed canopy (Credit: Stull, 1988)

ASL Wind Profile II

As you can see, we have introduced three more parameters to describe the velocity profile, so we will now briefly discuss their meaning:

Momentum roughness length (z_0): This parameter describes the roughness of the surface and can be thought of as the intercept of the log line when velocity goes to zero. Typical values are shown below and a general approximation is that $z_0 = 0.1h$, where h is the roughness or canopy height.

Zero-plane displacement (d): This is an offset to apply the log-law over a dense canopy. Studies have confirmed that a good estimate is $d = 0.7h$.

Friction velocity (u_*): Also called **shear velocity**, is an important velocity scale that is related to the surface shear stress by $u_* = \sqrt{\frac{\tau_0}{\rho_a}}$, where τ_0 is the surface shear stress.

Surface description	z_0 (m)
Large water surfaces ("average")	
Snow, mud flats	0.0001–0.0005
Smooth runways	
Short grass	0.008–0.02
Long grass, prairie	0.02–0.06
Short agricultural crops	0.05–0.10
Tall agricultural crops	0.10–0.20
Prairie or short crops with scattered bushes and tree clumps	0.20–0.40
Continuous bushland	
Bushland in rugged and hilly (50–100 m) terrain	1.0–2.0
Mature pine forest	0.80–1.5
Tropical forest	1.5–2.5
Fore-Alpine terrain (200–300 m) with scattered tree stands	3.0–4.0

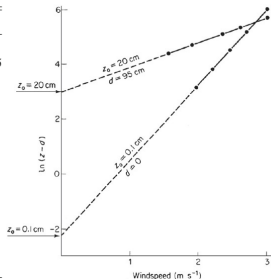


Figure 6: Typical z_0 values for different surfaces (left; Credit: Brusaert 2005) and a schematic showing the meaning of d and z_0 (right; Credit: Monteith and Unsworth, 2007)

ASL Wind Profile III

Logarithmic Law in Non-Neutral ASL:

Observations show that in the stable and unstable ASL, the velocity profile deviates from the log law as shown below. Now if we differentiate the original log law we get:

$$\bar{u} = \frac{u_*}{\kappa} \ln \left(\frac{z-d}{z_0} \right) \quad \Rightarrow \quad \frac{d\bar{u}}{dz} = \frac{u_*}{\kappa(z-d)},'$$

To account for the atmospheric stability or instability conditions, we can multiply the velocity profile by a correction factor $\phi_M(R_i)$, that can be explained as a function of the gradient Richardson number. Therefore, the corrected velocity profile can be explained as follows:

$$\frac{d\bar{u}}{dz} = \frac{u_*}{\kappa(z-d)} \phi_M(R_i).$$

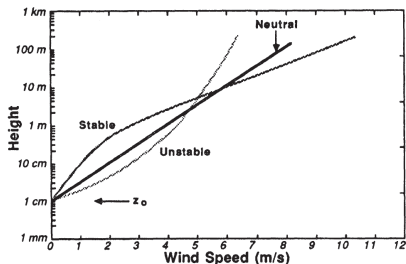


Figure 7: Impact of stability on mean velocity profile in the ASL (Stull, 1988).

ASL Wind Profile IV

Atmospheric (in)stability also affects the profile of moisture and temperature. There are several parameterizations that correct for the temperature ϕ_H and moisture ϕ_E profiles under non-neutral atmosphere. Here, we focus on the formulation by Dyer and Hicks (1970) as follows:

Correction factors for non-neutral atmosphere

$$\phi_M^2 = \phi_H = \phi_E = (1 - 16R_i)^{-0.5} = (1 - 16\zeta)^{-0.5} \quad R_i < 0 \quad \text{or} \quad \zeta < 0 \quad (\text{unstable})$$

$$\phi_M = \phi_H = \phi_E = \frac{1}{1 - 5R_i} = 1 + 5\zeta \quad 0 \leq R_i \leq 0.2 \quad \text{or} \quad \zeta > 0 \quad (\text{stable})$$

Since we have $\tau = \rho_a K_M d\bar{u}/dz$, $u_*^2 = \tau/\rho_a$ and $d\bar{u}/dz = \frac{u_*}{\kappa(z-d)} \phi_M$, we can conclude,

$$K_M = \kappa u_* (z - d) \phi_M^{-1}.$$

The same derivation holds for the eddy diffusivity of heat and moisture fluxes for non-neutral atmosphere as follows:

$$K_H = \kappa u_* (z - d) \phi_H^{-1}$$

$$K_E = \kappa u_* (z - d) \phi_E^{-1},$$

which can be used for computation of the turbulent heat and moisture fluxes under stable or unstable atmosphere. Obviously, the correction factors are equal to one for a neutral atmosphere.

ASL Evaporation and Heat Fluxes I

Moisture and Heat Fluxes in ASL:

As we have discussed earlier, the moisture and heat flux can be defined by the following turbulent diffusion equations:

$$E = -\rho_a K_E \frac{d\bar{q}_v}{dz} \quad [\text{Kg m}^{-2} \text{ s}^{-1}]$$

$$H = -\rho_a c_p K_H \frac{d\bar{T}}{dz} \quad [\text{W m}^{-2}]$$

where K_E and K_H are the respective eddy diffusivity values since the ASL is generally turbulent.

Now we would like to use these equations in combination with the information gained from the velocity profile to derive an equation for the fluxes in terms of variables we can easily measure.

Under neutral stability condition with zero displacement ($d = 0$, we have $K_H \simeq K_E \simeq K_M$ and $K_M \simeq \kappa z u_*$. As results one can have

$$E = -\rho_a (\kappa z u_*) \frac{d\bar{q}_v}{dz}$$

and thus

$$d\bar{q}_v = -\frac{E}{\rho_a \kappa u_*} \frac{dz}{z} \Rightarrow \int_{q_{v0}}^{q_v} d\bar{q}_v = \frac{-E}{\rho_a \kappa u_*} \int_{z_{0v}}^z \frac{dz}{z},$$

After integration, one can obtain

$$\bar{q}_v(z) - \bar{q}_{v0} = -\frac{E}{\rho_a \kappa u_*} \ln\left(\frac{z}{z_{0v}}\right),$$

ASL Evaporation and Heat Fluxes II

which can be rearranged as follows with substitution for $u_* = \kappa \bar{u}(z) / \ln(z/z_{0m})$:

$$E = \frac{\rho_a \kappa^2 \bar{u}(z)}{\ln\left(\frac{z}{z_{0v}}\right) \ln\left(\frac{z}{z_{0m}}\right)} (\bar{q}_{v0} - \bar{q}_v(z)),$$

z_{0m} : momentum surface roughness length (wind profile goes to zero)

z_{0v} : moisture surface roughness length (moisture profile goes to near surface \bar{q}_{v0})

\bar{q}_{v0} : near surface specific humidity of air

$\bar{q}_v(z)$: air specific humidity at elevation z (typically $z = 2$ m)

The above moisture flux is often expressed in a more compact and intuitive form as follows, where the displacement height is nonzero:

Land Surface Moisture Flux in Neutral Condition

$$E = \rho_a \frac{\bar{q}_{v0} - \bar{q}_v(z)}{r_{av}}$$
$$r_{av} = \frac{\ln\left(\frac{z-d}{z_{0m}}\right) \ln\left(\frac{z-d}{z_{0v}}\right)}{\kappa^2 \bar{u}(z)}$$

where r_{av} is called **aerodynamic vapor resistance** [$s\ m^{-1}$]. This is analogous to circuits with resistance to an electric current, where there is a voltage potential. Here, the potential is between specific humidity values.

ASL Evaporation and Heat Fluxes III

Similarly, we can also derive an equation for the sensible heat flux near the land surface as follows:

Land Surface Heat Flux in Neutral Condition

$$H = \rho_a c_p \frac{\bar{T}_0 - \bar{T}(z)}{r_{ah}}$$
$$r_{ah} = \frac{\ln\left(\frac{z-d}{z_{0m}}\right) \ln\left(\frac{z-d}{z_{0h}}\right)}{\kappa^2 \bar{u}(z)}$$

where r_{ah} is called **aerodynamic heat resistance** [s m^{-1}]. Estimating the values of z_{0v} and z_{0h} require profile data for moisture and heat. These roughness parameters tend to be more variable than z_{0m} . **Reynold's analogy allows us to assume $z_{0v} = z_{0h}$, and if there is no available heat or moisture profile data, then we may assume $z_{0v} = z_{0h} = 0.1z_{0m}$** (Allen et al. 2005). The reason for the difference is that rough surfaces absorb momentum more efficiently than heat and moisture.

Land Surface Heat/Moisture Flux in **Non-neutral** Condition (Thom 1975)

$$H = \rho_a c_p \frac{\bar{T}_0 - \bar{T}(z)}{r'_{ah}}$$
$$E = \rho_a \frac{\bar{q}_{0v} - \bar{q}_v(z)}{r'_{av}}$$

ASL Evaporation and Heat Fluxes IV

As we explained, the heat and moisture fluxes are increased (decreased) for unstable (stable) atmosphere and the correction factors are obtained through the $\phi(\cdot)$ functions.

Land Surface Heat/Moisture Flux in **Non-neutral** Condition (Thom 1975)

$$r'_{ah} = r_{ah} \times (\phi_H \phi_M)$$

$$r'_{av} = r_{av} \times (\phi_E \phi_M)$$

Land Surface Heat/Moisture Flux in **Non-neutral** Condition (Businger 1971)

$$r'_{ah} = \frac{\left[\ln \left(\frac{z-d}{z_{0m}} \right) - \psi_M(\zeta) \right] \left[\ln \left(\frac{z-d}{z_{0h}} \right) - \psi_H(\zeta) \right]}{\kappa^2 \bar{u}(z)}$$

$$r'_{av} = \frac{\left[\ln \left(\frac{z-d}{z_{0m}} \right) - \psi_M(\zeta) \right] \left[\ln \left(\frac{z-d}{z_{0v}} \right) - \psi_E(\zeta) \right]}{\kappa^2 \bar{u}(z)}$$

$$\psi_H(\zeta) = \psi_E(\zeta) = \begin{cases} -5\zeta & \zeta > 0 \text{ (stable)} \\ 2 \ln \left(\frac{1+x^2}{2} \right) & \zeta < 0 \text{ (unstable)} \end{cases}$$

$$\psi_M(\zeta) = \begin{cases} -5\zeta & \zeta > 0 \text{ (stable)} \\ 2 \ln \left(\frac{1+x}{2} \right) + \ln \left(\frac{1+x^2}{2} \right) - 2 \tan^{-1}(x) + \frac{\pi}{2} & \zeta < 0 \text{ (unstable)} \end{cases}$$

$$x = (1 - 16\zeta)^{0.25} \quad \text{note: } x \text{ is in radian.}$$

ASL Evaporation and Heat Fluxes V

In Summary: We defined moisture and heat fluxes near the soil in terms of more easily obtainable parameters as follows:

Evaporative Flux:

$$E = \rho_a \frac{\bar{q}_{v0} - \bar{q}_{va}}{r_{av}} \quad [\text{kg m}^{-2} \text{ s}^{-1}],$$

\bar{q}_{v0} : mean near surface specific humidity [kg-water kg-air⁻¹]

\bar{q}_{va} : mean air specific humidity [kg-water kg-air⁻¹]

r_{av} : aerodynamic vapor resistance [s m⁻¹]

Note: By definition, the **latent heat flux** is $LE = \mathcal{L}_{\ell v} E$ [W m⁻²].

Sensible Heat Flux:

$$H = \rho_a c_p \frac{\bar{T}_0 - \bar{T}_a}{r_{ah}} \quad [\text{W m}^{-2}],$$

\bar{T}_0 : mean surface virtual potential temperature [K]

\bar{T}_a : mean near surface air temperature [K]

r_{ah} : aerodynamic heat resistance [s m⁻¹].

To estimate the above fluxes, we need:

- ▷ **Meteorological state variables:** \bar{u} , T_a , and q_{va} —from models or observations.
- ▷ **Surface state variables:** T_0 and q_0 —from models or observations.
- ▷ **Surface parameters:** e.g., z_{0m} , z_{0h} , z_{0v} , the displacement height (d)—from observations and calibration studies.

To reiterate, typically we assume $r_{ah} = r_{av}$ and that $z_{0v} = z_{0h} = 0.1z_{0m}$, while $z_{0m} = 0.1h$ when do not have any other prior data.

ASL Evaporation and Heat Fluxes VI

Soil Moisture Effects on ET Flux:

For a saturated soil surface or open water, **the evaporation is at its potential rate (E_p)** and it is reasonable to assume that $q_{v0} = q_{vs}$ (saturated specific humidity). However, the soil moisture water deficit and its implications on the rate of evaporation fluxes may be characterized as follows:

$$E = \beta(\theta)E_p$$

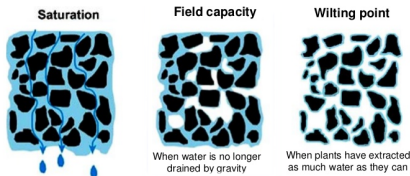
θ : soil moisture content [$\text{cm}^3 \text{ cm}^{-3}$]

$\beta(\theta)$: soil moisture correction multiplier, $0 \leq \beta(\theta) \leq 1$

$$\beta(\theta) = \begin{cases} 1 & \theta \geq \theta_{fc} \\ \frac{\theta - \theta_{wp}}{\theta_{fc} - \theta_{wp}} & \theta_{wp} \leq \theta \leq \theta_{fc} \\ 0 & \theta \leq \theta_{wp} \end{cases}$$

θ_{fc} : soil moisture content at field capacity [$\text{cm}^3 \text{ cm}^{-3}$]

θ_{wp} : soil moisture content at wilting point [$\text{cm}^3 \text{ cm}^{-3}$]



Canopy Effects on Evaporation I

Transpiration in ABL and ET fluxes

Plants synthesize visible solar energy (photosynthesis) for their metabolism. Throughout this process, water is evaporated by areal parts of the plant such as leaves, stems, etc. The process of water vaporization from soil to atmosphere through plant's metabolism is called **transpiration**. On plant's leaves there are pores called **stomata** that allow plants to uptake CO_2 for photosynthesis and release oxygen and water vapor to the atmosphere through the process of transpiration, which reduces the plants temperature. The process and rate of transpiration are controlled by the dynamics of stomata.

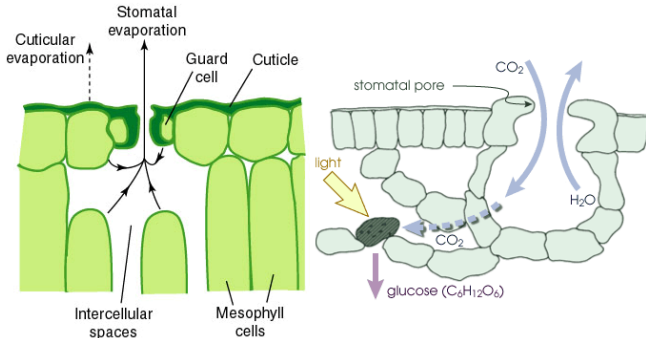


Figure 8: Pathway for water loss from surface of a leaf (Credit: Jones, 1983)

Canopy Effects on Evaporation II

Stomata respond to environmental forcings (e.g., air temperature, humidity, wind speed, sunlight intensity, soil water supply, etc.) by changing their opening size. When plants are under stress, typically stomata close their aperture to protect the plants against excessive water loss and wilting. As a general rule, larger leaves have more stomata which increases the ET rate from the leaves. Some plants have a waxy cuticle that reduces ET from the leaf surface (e.g., Xerophyte plants) such as the Cactus family. As noted, environmental stress affects plant's transpiration rate as follows:

- Increase of light (net radiation) increases the transpiration up to an asymptotic limit
- Increase of temperature increases the transpiration up to a threshold
- Decrease of the relative humidity generally increases the rate of transpiration
- Increase of soil moisture increases the rate of transpiration

Note that from a mass transport point of view, moisture gradient is the main driver of the plant's transpiration. Inside of the plant's leaf, the air is saturated, while it is likely that the outside air is sub-saturated.

In general, plants naturally attempt to increase their survivability in response to environmental stresses. The guard cells that control the size of the stomatal openings play a very critical and complex regulatory role. For instance when temperature increases above the tolerance limits of the plant, the stomatal openings start to reduce their sizes to reduce the rate of evaporation and increase the chance of survival. This complex regulatory role is often simplified and parameterized through the canopy resistance factor.

The canopy resistance can be modeled based on the stomatal resistance as follows:

Canopy Effects on Evaporation III

Canopy resistance r_c :

$$r_c = \frac{r_s}{\text{LAI}} \quad [\text{s}^{-1} \text{ m}]$$

r_s : stomatal resistance $[\text{s}^{-1} \text{ m}]$ r_c : canopy resistance $[\text{s}^{-1} \text{ m}]$ LAI: Leaf Area Index [-]

LAI is the one-sided green leaf area per unit ground surface area and ranges from $0 < \text{LAI} < 10$, where the upper bound refers to dense conifer forests.

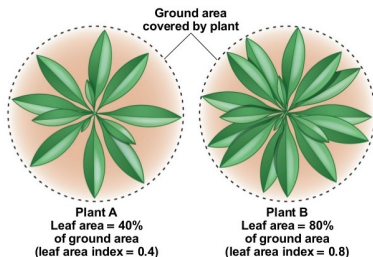


Figure 9: Schematic showing the concept of the Leaf Area Index (LAI).

As mentioned, there are many forcings that impact stomatal behavior and thus the r_s . An example of the existing equations is the following:

$$r_s = \frac{r_{min}}{f_{R_s} \cdot f_{T_a} \cdot f_{\delta_e} \cdot f_{\theta}}$$

Canopy Effects on Evaporation IV

where r_{min} is the minimal stomatal resistance determined from experiments on the plant. The forcing parameters (f) may be parameterized as follows (Margulis, 2016):

Forcings affecting the stomatal resistance r_s

Radiation: $f_{R_s} = \frac{1.105R_{is}}{1.007R_{is}+104.4}$, where R_{is} is incident shortwave radiation [$W m^2$]

Air Water Vapor Deficit: $f_{\delta_e} = 1 - 0.00023\delta_e$, where $\delta_e = e_s(T_a) - e(T_a)$ [$N m^{-2}$]

Temperature: $f_{T_a} = \frac{T_a(40-T_a)^{1.18}}{690}$, where $0 \leq T_a \leq 40$ [$^{\circ}C$]

Soil Moisture: $f_{\theta} = \begin{cases} 1 & \theta \geq \theta_{fc} \\ \frac{\theta - \theta_{wp}}{\theta_{fc} - \theta_{wp}} & \theta_{wp} \leq \theta \leq \theta_{fc} \\ 0 & \theta \leq \theta_{wp} \end{cases}$ where θ [$cm^3 cm^{-3}$]

Canopy Effects on Evaporation V

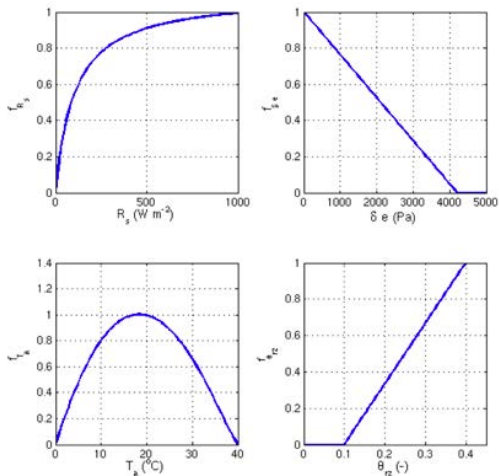


Figure 10: Examples of forcing functions based on equations shown above (Margulis, 2016). When f factors increase the resistance decrease and thus the transpiration flux increase and vice versa.

Canopy Effects on Evaporation VI

With this in mind, we can now consider that the overall resistance to evaporation is a combination of some canopy resistance (r_c), that accounts for the various vapor transport mechanisms within the canopy and soil surface and the aerodynamic vapor resistance (r_{av}) that accounts for the effects of the wind velocity and turbulent transport. Since these resistances are in series, we can sum them into a single term:

$$r_{ac} = r_c + r_{av} \quad [s \text{ m}^{-1}]$$

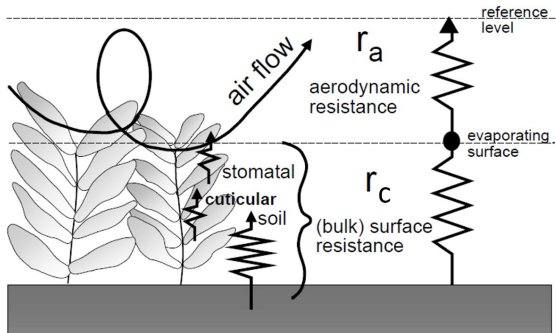


Figure 11: Simplified schematic of resistances to evaporation from a plan canopy. (Adapted from Allen et al. 1998)

Modeling ET I

Energy Balance Models

So far, we have focused primarily on flux-based or aerodynamic methods for calculating sensible and latent heat fluxes. However, it is also very common to further constraint the calculate ET flux based on the surface energy balance (SEB).

$$R_n = LE + H + G$$

By convention, net radiation R_n , is positive when toward the surface, whereas H , LE , and G are positive away from the surface. Figure 12 shows the typical diurnal cycle of the energy balance.

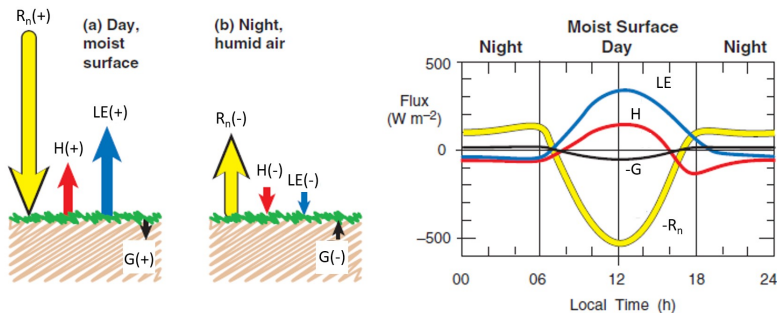


Figure 12: Schematic of typical diurnal variation of the surface energy balance for a well watered soil surface (Stull, 2015).

Modeling ET II

However, from the SEB, we have one equation and three unknowns, considering we can easily estimate or measure R_n . Since the ground heat flux does not vary significantly compared to the sensible and latent heat fluxes, we typically define the available energy that is partitioned to the sensible and latent heat fluxes as follows:

$$Q_n = R_n - G.$$

Experimental evidence suggests that we can assume G is some fraction of R_n . Based on the field data $G \approx 0.05R_n$ over vegetation canopies and $G \approx 0.315R_n$ over the bare soil. This assumption leaves us with one equation and two unknowns. Therefore, to solve the land surface energy balance equation, we only need an extra equation.

Bowen Ratio Method: To add an extra equation that enables us to obtain the sensible and latent heat flux, we can use the **Bowen ratio**, which is simply the ratio of sensible to latent heat flux:

$$\beta = \frac{H}{LE} = \frac{\rho_a c_p \frac{\bar{T}_{v0} - \bar{T}_{va}}{r_{ah}}}{\mathcal{L}_{lv} \rho_a \frac{\bar{q}_{v0} - \bar{q}_{va}}{r_{av}}} = \frac{c_p (\bar{T}_{v0} - \bar{T}_{va})}{\mathcal{L}_{lv} (\bar{q}_{v0} - \bar{q}_{va})} = \frac{c_p P (\bar{T}_{v0} - \bar{T}_{va})}{0.622 \mathcal{L}_{lv} (\bar{e}_0 - \bar{e}_a)}$$

Note that in the above expansion, we used $q_v = \epsilon e/P$. Therefore, we can calculate the Bowen ratio using easily measured water vapor pressure rather than the specific humidity. In order for us to cancel out terms in the above equation, we had to assume $r_{ah} = r_{av}$. Some typical values for β are:

$$\beta \simeq 5 \text{ semi-arid} \quad \beta \simeq 0.5 \text{ grassland} \quad \beta \simeq 0.2 \text{ irrigated agriculture} \quad \beta \simeq 0.1 \text{ open water}$$

It is clear that the Bowen ratio decreases over moist surfaces as most energy is going to evaporation. As explained, if the Bowen ratio is given and we have Q_n , one can use these two equations to estimate the sensible H and latent LE heat fluxes:

$$\begin{cases} Q_n = LE + H \\ H = \beta LE \end{cases} \Rightarrow LE = \frac{Q_n}{1 + \beta} \quad \text{and} \quad H = \frac{\beta Q_n}{1 + \beta}.$$

Modeling ET III

Penman-Monteith

Now we have covered both flux-based aerodynamic methods and an energy balance approach for calculating evaporative and sensible heat fluxes. The methods that combine these two, known as **combination methods**, are the best models of ET that we currently have simply because their solution is constrained to both flux models and land surface energy balance equation. The most popular of which is the **Penman-Monteith equation**. To derive this equation, we begin with the sensible heat flux as defined previously:

$$H = \rho_a c_p \frac{\overline{T}_0 - \overline{T}_a}{r_{ah}}$$

\overline{T}_0 : Air temperature at the surface \overline{T}_a : Air temperature at height of measurement

For the latent heat flux we also have,

$$LE = \mathcal{L}_{lv} \rho_a \frac{\overline{q}_{v0} - \overline{q}_{va}}{r_{av}} = \mathcal{L}_{lv} \rho_a \frac{\varepsilon(\overline{e}_{s0} - \overline{e}_a)}{r_{av} P}$$

where

e_{s0} : Saturation vapor pressure near the surface [Pa]

\overline{e}_a : Vapor pressure at height of measurement [Pa]

where we used $q_v = \varepsilon \frac{e}{P}$, e : water vapor pressure [Pa], P : air pressure [Pa], and $\varepsilon = 0.622$.

NOTE: One of the key assumptions to the Penman-Monteith approach is that the surface is saturated thus we assumed $e_0 = e_{s0}$. At this point, we are going to drop the overbar on all the variables for notational convenience. Just know we are referring to mean values of quantities as Penman-Monteith is typically used for daily and/or hourly mean values.

Modeling ET IV

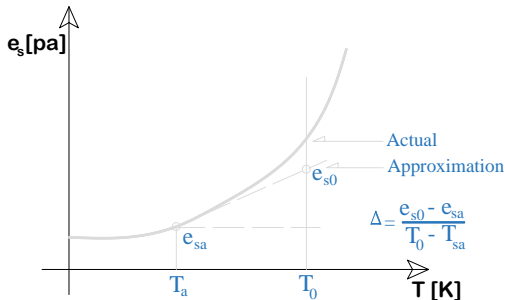


Figure 13: Linearization of the Clausius-Clapeyron equation around T_a .

Meteorological stations typically measure the air temperature and water vapor pressure at 2 meters above the surface and not near the soil surface. This is why Penman sought a way in 1948 to determine ET from air measurements above the surface (e.g., T_a at 2 m). Penman decided to use a first-order Taylor approximation (see Figure 13) to linearize the Clausius-Clapeyron equation around T_a for relating the unknown e_{s0} to the measured e_{sa} as follows:

$$e_{s0} \simeq e_{sa} + \left. \frac{\partial e_s}{\partial T} \right|_{T=T_a} (T_0 - T_a)$$

where e_{sa} is the saturation vapor pressure at T_a . As previously explained, the Clausius-Clapeyron equation follows an exponential form and thus its derivative can be explained as follows:

Modeling ET V

Use of the Clausius-Clapeyron in Penman-Monteith

$$\text{Clausius-Clapeyron: } e_s(T) = 6.11 \exp \left[-\frac{\mathcal{L}_{\ell v}}{R_v} \left(\frac{1}{T} - \frac{1}{273.15} \right) \right] \text{ [hPa]}.$$

$$\mathcal{L}_{\ell v}: \text{ latent heat of vaporization} = 2.5 \times 10^6 \text{ [J kg}^{-1}\text{]}$$

$$R_v: \text{ water vapor gas constant} = 461 \text{ [J kg}^{-1} \text{K}^{-1}\text{]}$$

Note that in the above equation 6.11 [hPa] and $T = 273.15$ [K] refer to the triple point pressure and temperature, which is used as an arbitrary saturation point. In other words, the CC equation in its original form is $e_s(T_0) = e_s(T_a) \exp \left[-\frac{\mathcal{L}_{\ell v}}{R_v} \left(\frac{1}{T_0} - \frac{1}{T_a} \right) \right]$, thus

$$\text{Slope of Clausius-Clapeyron: } \Delta = \left. \frac{\partial e_s}{\partial T} \right|_{T=T_a} = \frac{\mathcal{L}_{\ell v}}{R_v} \cdot \frac{e_{sa}}{T_a^2} \text{ [Pa K}^{-1}\text{]}$$

$$\text{Taylor Approximation used in Penman Monteith Derivation: } \Delta = \frac{e_{s0} - e_{sa}}{T_0 - T_a}$$

From here, let us now substitute for $e_{s0} = e_{sa} + \Delta(T_0 - T_a)$ into the equation for LE :

$$LE = \mathcal{L}_{\ell v} \rho_a \frac{\varepsilon(e_{sa} - e_a + \Delta(T_0 - T_a))}{r_{av} P}$$

Modeling ET VI

and then substitute in for $(T_0 - T_a)$ using the equation for H :

$$LE = \mathcal{L}_{\ell v} \rho_a \frac{\varepsilon(e_{sa} - e_a + \Delta H \frac{r_{ah}}{\rho_a c_p})}{r_{av} P}$$

Now we will clean up the expression by defining the **psychrometric constant** as $\gamma = \frac{P c_p}{\varepsilon \mathcal{L}_{\ell v}}$ and $\gamma^* = \gamma \cdot \frac{r_{av}}{r_{ah}}$ which will yield:

$$LE = \frac{\rho_a c_p}{\gamma^* r_{ah}} (e_{sa} - e_a) + \frac{\Delta}{\gamma^*} H$$

Thus far, we have only used aerodynamic equation, so let us use the information content of the energy balance equation by substituting $H = Q_n - LE$:

$$LE = \frac{\rho_a c_p}{\gamma^* r_{ah}} (e_{sa} - e_a) + \frac{\Delta}{\gamma^*} (Q_n - LE)$$

From here, we now want to solve for LE as follows:

$$(1 + \frac{\Delta}{\gamma^*}) LE = \frac{\rho_a c_p}{\gamma^* r_{ah}} (e_{sa} - e_a) + \frac{\Delta}{\gamma^*} Q_n$$

Multiply the whole equation by γ^* :

$$(\Delta + \gamma^*) LE = \frac{\rho_a c_p}{r_{ah}} (e_{sa} - e_a) + \Delta Q_n$$

Modeling ET VII

Use the relation $\frac{c_p}{r_{ah}} = \frac{\varepsilon \mathcal{L}_{lv}}{P} \frac{\gamma^*}{r_{av}}$

$$(\Delta + \gamma^*)LE = \gamma^* \rho_a \mathcal{L}_{lv} \frac{\varepsilon}{P} \frac{(e_{sa} - e_a)}{r_{av}} + \Delta Q_n$$

and finally using the relation $q_v = \varepsilon \frac{e}{P}$ and solving for LE , we obtain the original Penman equation:

Penman Equation (1948)

$$LE = \frac{\gamma^*}{\Delta + \gamma^*} \rho_a \mathcal{L}_{lv} \frac{(q_{sa} - q_a)}{r_{av}} + \frac{\Delta}{\Delta + \gamma^*} Q_n \quad \text{where} \quad \gamma^* = \gamma \left(\frac{r_{av}}{r_{ah}} \right)$$

One can clearly see that in this equation the term on the LHS is the **air moisture deficit** while the second term is the **available energy for evaporation**.

However, Penman's original equation could be applied only over water, bare soil, and short vegetation as it assumes that the surface is completely saturated. The equation also does not account for plant transpiration. Therefore, one of Penman's student's, John Monteith modified the original equation to account for the effect of the canopy on ET. As mentioned previously, the actual resistance to vapor pressure is:

$$r_{ac} = r_c + r_{av} \quad [\text{s m}^{-1}]$$

as shown in Figure 11. Now if we just update the value of γ^* in the original Penman equation as follows:

$$\gamma^* = \gamma \cdot \frac{r_{av}}{r_{ah}} := \gamma \cdot \frac{r_{av} + r_c}{r_{ah}} \simeq \gamma \left(1 + \frac{r_c}{r_{ah}} \right)$$

Modeling ET VIII

with the assumption that $r_{ah} = r_{av}$, we have the Penman-Monteith equation, which has been the most widely used method for computation of ET fluxes in the past decades. Typically, for brevity, we could assume $r_{ah} = r_{av} = r_a$. Using this, the final form of the Penman-Monteith equation is:

Penman-Monteith Equation (1965)

$$LE = \frac{\gamma^*}{\Delta + \gamma^*} \rho_a \mathcal{L}_{lv} \frac{(q_{sa} - q_a)}{r_a} + \frac{\Delta}{\Delta + \gamma^*} Q_n \quad \text{where} \quad \gamma^* = \gamma \left(1 + \frac{r_c}{r_a} \right)$$

Therefore, ET can be estimated if one has the required measurements, namely net radiation, air temperature, air humidity, wind speed as well as estimates for the roughness lengths and canopy resistance. We have already covered a detailed way to calculate r_c ; however a table of typical values is given below.

Vegetation type	r_c (s m ⁻¹)
Temperate grassland	60
Coniferous forest	50
Temperate deciduous forest	50
Tropical rain forest	80
Cereal crops	30
Broadleaved herbaceous crops	35

Figure 14: Typical values of canopy resistance, r_c (Credit: Monteith and Unsworth, 2007).

Modeling ET IX

FAO Standardized Penman-Monteith Equation

There have also been attempts to further simplify the Penman-Monteith equation for more practical use. In 1998, the Food and Agriculture Organization (FAO) of the UN put out a simplified standardized method of the equation as follows:

$$ET_o = \frac{0.408 \Delta (R_n - G) + \gamma \frac{900}{T + 273} u_2 (e_s - e_a)}{\Delta + \gamma(1 + 0.34 u_2)}$$

where	ET_o	reference evapotranspiration [mm day^{-1}],
	R_n	net radiation at the crop surface [$\text{MJ m}^{-2} \text{day}^{-1}$],
	G	soil heat flux density [$\text{MJ m}^{-2} \text{day}^{-1}$],
	T	mean daily air temperature at 2 m height [$^{\circ}\text{C}$],
	u_2	wind speed at 2 m height [m s^{-1}],
	e_s	saturation vapour pressure [kPa],
	e_a	actual vapour pressure [kPa],
	$e_s - e_a$	saturation vapour pressure deficit [kPa],
	Δ	slope vapour pressure curve [$\text{kPa } ^{\circ}\text{C}^{-1}$],
	γ	psychrometric constant [$\text{kPa } ^{\circ}\text{C}^{-1}$].

Figure 15: FAO Standardized Penman-Monteith Equation (Credit: Allen et al. 1998).

Modeling ET X

This equation is calculating the daily ET of a standardized reference crop which is a well-watered short grass (0.12 m tall). Additionally, the equation assumes measurements are all at 2 m height. Then using the reference ET_0 , one can obtain the daily crop ET_c values as follows:

$$ET_c = K_c ET_0$$

where K_c is a crop coefficient that accounts for the difference from the reference crop <http://www.fao.org/docrep/X0490E/x0490e0b.htm>. Various values of K_c have been recorded for differing crops under various climatic conditions and growth stages (Figure 17).

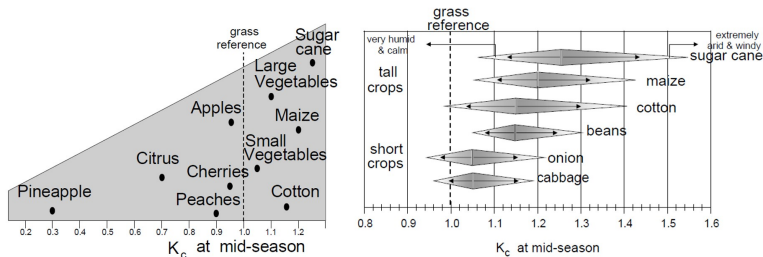


Figure 16: Mean values of K_c for fully grown crops (left) and the range of its variability (right) under varying climatic conditions (Credit: Allen et al.1998). The upper bounds represent extremely arid and windy conditions, while the lower bounds are valid under very humid and calm weather conditions.

Modeling ET XI

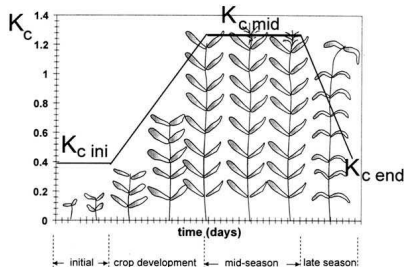


Figure 17: Typical changes of the crop coefficient throughout the growth season.

There are formulas to construct the evolution of the crop coefficient at this [FAO web page](#) [link].

Priestley-Taylor Equation

If meteorological data of humidity and wind velocity are not available, an even simpler form of Penman-Monteith is the Priestley Taylor equation:

$$LE = \alpha \frac{\Delta}{\Delta + \gamma} Q_n$$

where field experiments found for well watered fields, on average $\alpha \approx 1.2 - 1.3$. As you can see, this equation essentially states that the aerodynamic component of the Penman-Monteith accounts for 30 percent of the total ET in a well-watered condition.