CEGE 4501 Hydrologic Design

Chapter 3:
Solar Radiation and Surface Energy Balance

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Driven to Discover℠

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Outline

Radiative Energy I

Heat energy can be **transferred** via different mechanisms including:

Conduction: Transfer of a conserved quantity (e.g., energy or mass of a substance) through **molecular collisions or diffusion**. This is the main mechanism in transfer of heat energy in solid materials.

Advection: Transfer of a conserved quantity (e.g., energy or mass of a substance) via **bulk displacement of matter**, which can take place in fluids or soft solids. In general, any substance or conserved, extensive quantity can be advected by a fluid that can hold or contain that quantity or substance.

Convection: Transfer of a conserved quantity via **bulk displacement of matter and turbulent diffusion** (Advection + diffusion). Convection, can take place in fluids and soft solids or mixtures where particles can move past each other. Turbulent diffusion is much stronger than molecular diffusion and is due to random and chaotic swirls of motions of fluid particles. When a flow is turbulent, molecular diffusion is negligible compared to the turbulent diffusion.

Radiation: Transfer of **heat energy** by means of electromagnetic waves, with or without any intervening physical medium.

The Sun is the main source of heat energy behind hydrologic, atmospheric, and oceanic circulation. Because of the differential radiative heating and thus temperature gradient from equators to poles, heat fluxes are formed to balance the uneven distribution of heat. As we discussed, this differential heating generates large-scale pressure gradient and thus circulation patterns from equators to pole and vice versa that the drives the hydrologic cycle. These cells (e.g., Hadly Cells or Ferrel Cells) are often called convective cells because convection is the main mechanism behind the large-scale transport of heat and water vapor in the Earth's atmosphere.

Radiative Energy II

Electromagnetic waves are formed by elementary particle, called photons, travel at the speed of light at different wavelengths λ [m]:

$$\lambda = cT,$$

where $c = 3 \times 10^8$ [m s^{-1}] is the speed of light, T is the wave period [s^{-1}]. Often, we express $\lambda\nu = c$, where $\nu = 1/T$ is the frequency in Hertz [Hz].

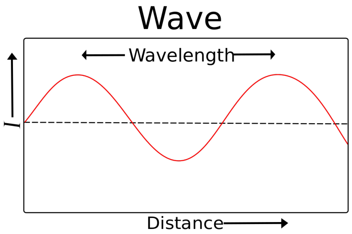

Figure 1: Wavelength is the distance between two crests of a wave.

The electromagnetic energy (EM) spreads across different wavelengths forming a spectrum shown in the following figure.

Radiative Energy III

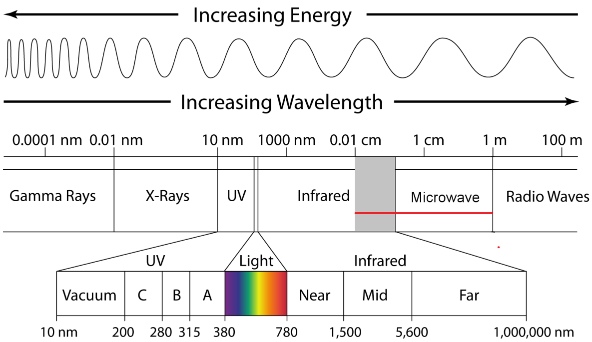

Figure 2: Electromagnetic (EM) spectrum

The wavelength of interests for hydrologists may include ultra-violet, visible, infrared, and microwave bands.

It is worth nothing that almost 50% of the earth heating is coming from the Sun's radiative energy in infrared bands, which has important implications on global warming.

Radiative Energy IV

To properly quantify the EM radiative energy we first need to define some definitions and terminology as follows:

Radiance: The radiant energy coming from a specific direction passing through a unit area perpendicular to that direction per unit time [$Wm^{-2}sr^{-1}$], where steradian [sr] is the unit of solid angle see Figure 3. Spectral radiance is the radiance per wavelength [$Wm^{-2}sr^{-1}m^{-1}$] or frequency [$Wm^{-2}sr^{-1}Hz^{-1}$].

Irradiance: The radiant energy passing through a unit area per unit time integrated over all solid angles [Wm^{-2}]. Similarly, spectral irradiance is the irradiance per unit wavelength [$Wm^{-2}m^{-1}$] or frequency [$Wm^{-2}Hz^{-1}$].

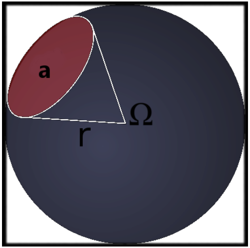

Figure 3: Schematic of the solid angle and its unit (steradian). A steradian is the solid angle facing an area of $a = r^2$. Clearly, a sphere is 4π steradians.

Some Radiation Laws I

A blackbody is a hypothetical physical object that **absorbs all** incident EM radiation.

Planck's law: The emitted radiation by a blackbody at temperature T [K], in terms of spectral radiances, as a function of wavelength (λ) or frequency (ν) is expressed as follows:

$$B_\lambda = \frac{2hc^2}{\lambda^5\left(e^{\frac{hc}{\lambda \kappa_B T}} - 1\right)} \quad [Wm^{-2}sr^{-1}m^{-1}] \quad \text{or} \quad B_\nu = \frac{2h\nu^3}{c^2\left(e^{\frac{h\nu}{\kappa_B T}} - 1\right)}[Wm^{-2}sr^{-1}\,Hz^{-1}],$$

where $\kappa_B = 1.38064 \times 10^{-23}$ [J K^{-1}] is the **Boltzmann constant** and $h = 6.626 \times 10^{-34}$ [J s^{-1}] represents the **Planck's constant**.

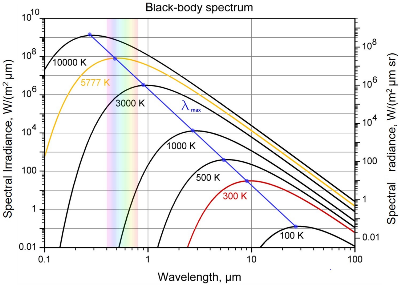

Figure 4: Planck's law as a function of temperature and wavelength. Note that the y-axis is in logarithm scale. The yellow and red curves denote the law for average temperature of the Sun and Earth. The blue line shows the Wien's displacement law explained below.

Some Radiation Laws II

Wien's displacement law: explains the wavelengths over which the Plank's law is maximum as a function of temperature T:

$$\frac{\partial B_\lambda}{\partial \lambda} = 0 \Rightarrow \lambda_{\text{max}} = \frac{2897}{T} \quad [\mu\text{m}],$$

The **Stefan-Boltzmann law** can be obtained by integrating the Planck's law in irradiance units ($4\pi B_\lambda$) over all wavelengths or frequencies as follows:

$$B = \int_\lambda 4\,\pi B_\lambda d\lambda = \sigma T^4. \quad [\text{Wm}^{-2}]$$

As a result, one can see that the Stefan-Boltzmann law describes radiative energy as **irradiance across all wavelengths**, where $\sigma = 5.67037 \times 10^{-8}$ [$\text{Wm}^{-2}\text{K}^{-4}$] is the Stefan-Boltzmann constant.

Graybody: Unlike a blackbody, natural objects are gray bodies, as they only absorb and thus emit a fraction of incident radiation. Radiation of a graybody is different than a blackbody up to a proportionality constant called the emissivity:

$$R_\lambda = \epsilon_\lambda B_\lambda,$$

where the dimensionless coefficient $0 < \epsilon_\lambda < 1$ is called spectral emissivity. As is clear, emissivity is an spectral quantity, which varies at different wavelengths so one can integrate spectral emissivity over all wavelengths $\epsilon = \int_\lambda \epsilon_\lambda d\lambda$ to obtain the so-called broadband emissivity. As a result, the Stefan-Boltzmann law for a graybody can be expressed as follows:

$$R = \epsilon\sigma T^4. \quad [\text{Wm}^{-2}]$$

Kirchhoff's Law: For an object with constant temperature, the absorbed radiative energy is equal to the emitted radiative energy. Therefore, absorptivity and emissivity are equal $a_\lambda = \epsilon_\lambda$, when an object is in thermodynamic equilibrium.

Radiation Partitioning I

In study of earth systems, we typically divide the EM spectrum into shortwave and longwave segments. The main reason is that the temperature of the Sun and the Earth are drastically different and thus the wavelengths of their radiation. According to the Wien's displacement law we have:

$$T_{sun} \approx 6000^{\circ} K \rightarrow \lambda_{max} \approx 0.48\ \mu m,$$
$$T_{earth} \approx 300^{\circ} K \rightarrow \lambda_{max} \approx 10\, \mu m.$$

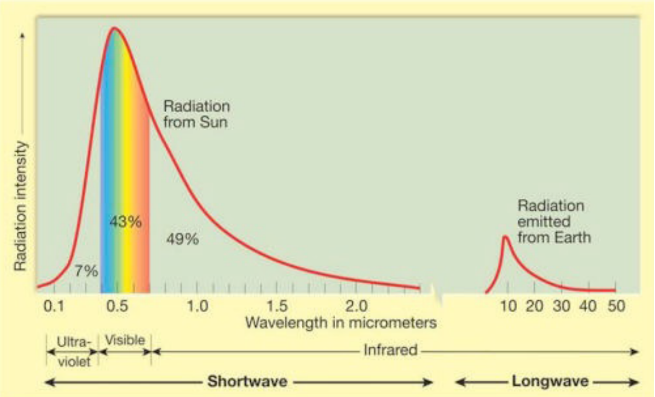

Figure 5: Solar shortwaves ($0.1 \leq \lambda \leq 4\mu m$) versus terrestrial longwaves ($4 \leq \lambda \leq 200\mu m$).

Radiation Partitioning II

It can be shown that the solar radiation above 4 μm is almost negligible. Therefore, by convention, we have:

$$\textbf{Shortwave radiation} = \int_{0.1}^{4\mu m} R_\lambda \, d\lambda \quad \textbf{Solar},$$

$$\textbf{Longwave radiation} = \int_{4}^{200\mu m} R_\lambda \, d\lambda \quad \textbf{Terrestrial} \quad [\mathrm{Wm^{-2}}].$$

Shortwave radiation contains : ultra-violet + visible + near-infrared + mid-infrared

Longwave radiation contains: mid-infrared + far-infrared

Radiation Budget:

Any incident radiation will be absorbed, scattered, or transmitted by nearby mass.

Absorption is a process that removes the radiant energy from an EM field and transfers it into the internal energy of the absorber.

Scattering or Reflection is a process that does not remove the energy from the EM field, but redirects it. Scattering often occurs through a volume (atmospheric depth). However, reflection occurs at the interface of two materials that could be **diffuse** and/or **specular** (see Figure 7)

Extinction/Attenuation is absorption + scattering.

Transmission is all radiant energy that passes through a medium without any attenuation.

Radiation Partitioning III

The **Radiative Energy Balance** at an interface can be written as:

$$R_\lambda = \mathcal{A}_\lambda + \mathcal{S}_\lambda + \mathcal{T}_\lambda \quad \Rightarrow \quad \frac{\mathcal{A}_\lambda}{R_\lambda} + \frac{\mathcal{S}_\lambda}{R_\lambda} + \frac{\mathcal{T}_\lambda}{R_\lambda} = 1 \quad \Rightarrow \quad a_\lambda + s_\lambda + \tau_\lambda = 1,$$

where a_λ is absorptivity, s_λ is reflectivity and τ_λ denotes transmissivity.

Clearly, the radiative balance holds for both short and longwave radiation as follows:

$$a_s + s_s + \tau_s = 1 \quad \text{(shortwave)}$$
$$a_l + s_l + \tau_l = 1 \quad \text{(longwave)}$$

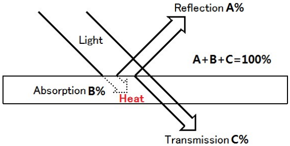

Figure 6: Absorption, Scattering/Reflection, and Transmission.

Often some of the terms in the above radiation balance equation are negligible:

Within the atmosphere longwave scattering is often negligible during the clear sky condition, hence, $a_l + \tau_l = 1$ (see, Figure 8).

At the Earth's surface both long and shortwave transmissivity are negligible. Therefore, for shortwave radiation, we have $a_s + s_s = 1$. The shortwave reflection at the Earth surface is called **albedo** and denoted by α in hydrologic community, which leads to $\alpha = 1 - a_s$.

Radiation Partitioning IV

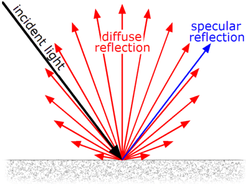

Figure 7: Diffused versus specular reflection.

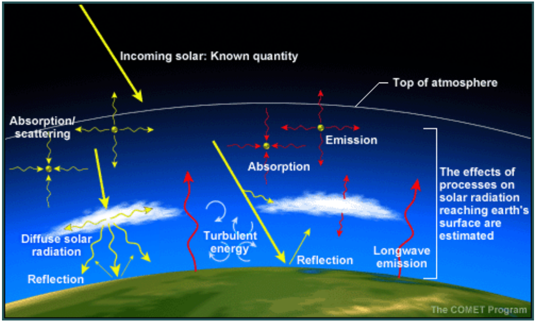

Figure 8: Schematic of absorption and scattering in atmosphere and earth's surface.

Solar radiation at top of the atmosphere I

To understand hydrologic cycle, it is key to know that how much of the Sun's energy will reach to the earth's surface and what will be its distribution. Using the Stefan-Boltzmann law, we have:

$$R = \sigma T^4 = 5.67 \times 10^{-8} \times 5780^4 = 6.33 \times 10^7 \quad [\mathrm{W\,m^{-2}}].$$

In far-field radiation, basically it is reasonable to assume that the radiation beams are parallel. Therefore, given that:

d=1.496×10^6 [km] (Sun-Earth distance)
r_s=696,300 [km] (Sun radius)
r_e=6371 [km] (Earth radius),

Assuming that $r_e \ll r_s \ll d$, one can easily compute the intensity of solar radiative energy at top of the atmosphere as follows:

$$\textbf{Solar Constant} = I_{sc} \approx 6.33 \times 10^7 \times \frac{r_s^2}{d^2} \approx 1367 \quad [\mathrm{W\,m^{-2}}]$$

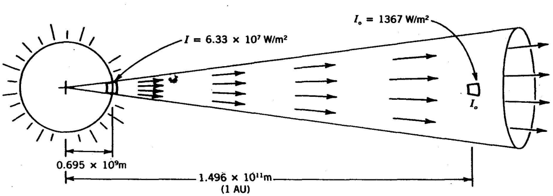

Figure 9: Far field solar radiation and solar constant at TOA.

Solar radiation at top of the atmosphere II

Note: recent detailed calculation by NASA using satellite observations shows that the solar constant varies between 1361-1362 [$\mathrm{Wm^{-2}}$] at the top of the atmosphere. However, it is still customary to use 1367 [$\mathrm{W\,m^{-2}}$] in hydrologic analysis as the solar constant.

Question: Why does the earth surface receive a non-uniform amount of radiative energy?

Answers:

- Earth is an ellipse (not a flat plane) and rotates around its central axis passing through its poles.
- Earth's axis of rotation is tilted by 23.5° with respect to its orbital plane, which is the main reason for experienced seasonality in the earth's climate.
- Earth's orbit is slightly elliptical and thus the earth's distance to the sun is variable throughout the year.

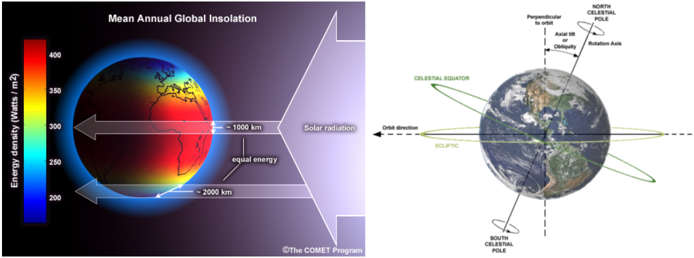

Figure 10: Schematic of global insolation and effects of earth geometry on distribution of the solar radiation on earth surface (left). The earth axis of rotation is tilted with respect to its orbital plane, which is in fact the main reason for seasonality of earth's climate.

Solar radiation at top of the atmosphere III

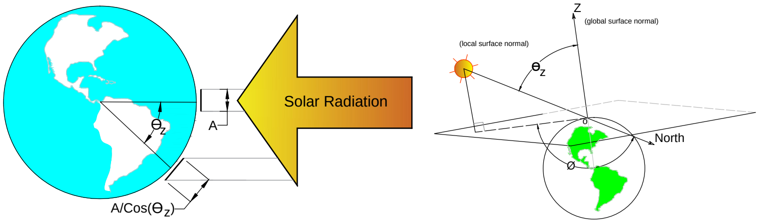

Figure 11: Schematic of global insolation and effects of earth spherical geometry on distribution of the solar radiation on earth surface. The solar zenith angle (θ_z), encodes the reduction on the intensity of radiation over higher latitudes, while ϕ is the so-called solar azimuth angle, which determines the planar orientation of solar incident radiation with respect to north.

Insolation (R_s) is the intensity of Sun's radiative energy received at top of the atmosphere that accounts for the fact that the incident radiation beams are not always perpendicular to the surface:

$$\text{Total power} = \text{intensity} \times \text{area} = I_{sc} \times A = R_s \times \frac{A}{\cos\theta_z},$$

thus $R_s < I_s$ and the solar insolation is,

$$R_s = I_{sc} \cos\theta_z .$$

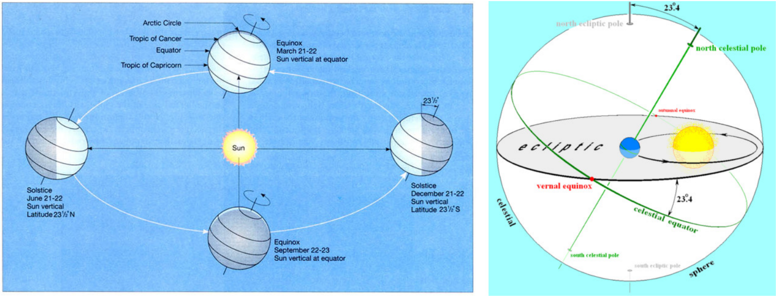

Figure 12: Summer and winter solstices and equinoxes.

Summer Solstice (June 20 or 21) refers to the longest day in the Northern Hemisphere. As is evident, summer solstice occurs while the Earth Northern Hemisphere is at its closest distance to the Sun and receives its maximum annual daily radiation budget. Analogously, while the Northern Hemisphere is at its longest distance with the Sun throughout the year, it experiences the longest night of the year. The day of Dec 21 or 22 is the **Winter Solstice** in the Northern Hemisphere while it is summer solstice in the Southern Hemisphere. Note that while it is summer solstice in the Northern Hemisphere, it is winter solstice in the Southern Hemisphere with the longest night of the year.

When the Earth's celestial/equatorial plane passes through the center of the Sun, the day and night are equal. These days (Sep 22 or 23 or Mar 20 or 21) are called **Equinoxes**.

Net Radiation at Earth's Surface I

In order to understand hydrologic fluxes at the surface, we need to know that how solar radiation partitions between the Earth's surface and its atmosphere. This partitioning is dynamic depending on the earth rotation and atmospheric composition such as cloud cover. Recent analysis of the attenuation of the incoming solar radiative energy within the Earth's atmosphere shows that earth has an **average** planetary albedo of around 30%, while nearly 23% of solar insolation is absorbed by the atmosphere and clouds, leaving around 47% of the insolation reaching the Earth's surface (Trenberth, 2009).

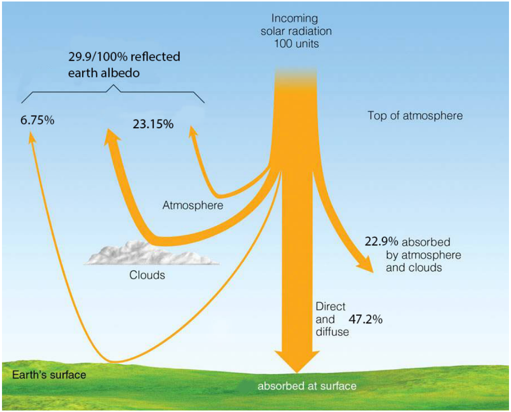

Figure 13: Estimates of planetary radiation attenuation (Trenberth,2009)

Net Radiation at Earth's Surface II

More specifically, we must determine the net input/output of short and longwave radiation at the Earth's surface, known as the **Net Radiation** [Wm^{-2}]

$$R_n = R_{is}(1-\alpha) + \epsilon_o R_{il} - R_{ol},$$

where R_n is the net radiation at he Earth's surface, R_{is} is the incoming shortwave radiation, α is the average earth surface albedo, ϵ_0 is the longwave surface emissivity, R_{il} is the incoming longwave radiation, and R_{ol} is the outgoing longwave radiation.

There are many empirical relations to estimate R_{is} based on location, season, and atmospheric condition, while the longwave components can be estimated using the Stefan-Boltzmann law

$$R_{il} = \epsilon_a \sigma T_a^4 \qquad R_{ol} = \epsilon_0 \sigma \ T_0^4$$

where T_a and T_0 represent the average air and surface temperatures in Kelvin.

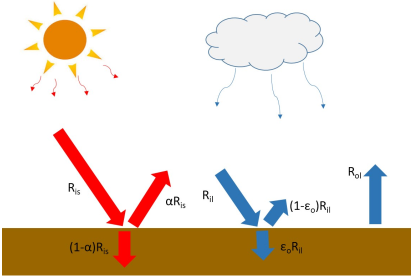

Figure 14: Schematic of shortwave and longwave radiation budget at Earth's Surface.

Net Radiation at Earth's Surface III

The net radiation at the earth surface is partitioned into three components to create the **Surface Energy Balance (SEB)**:

$$R_n = LE + H + G \qquad [\mathrm{W\,m^{-2}}]$$

where LE is the latent heat flux, H is the sensible heat flux, and G is the ground heat flux. We will discuss these very important components in the lessons on evaporation. However, as way of review, below is a figure of the estimated global energy budget containing the components discussed in this section.

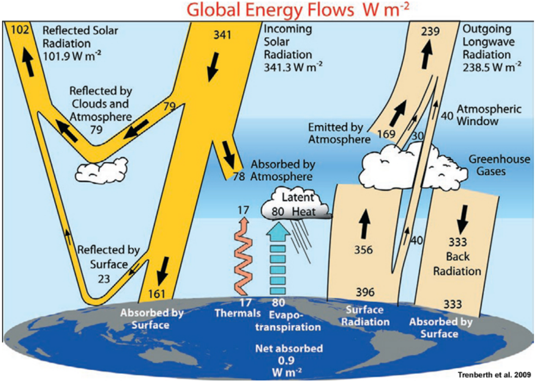

Figure 15: The global annual mean Earth's energy budget for 2000 to 2005 [$\mathrm{W\,m^{-2}}$] (from Trenberth et al. 2009).